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Interference and the Law of Energy Conservation

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Introductory physics textbooks consider interference to be a process of redistribution of energy from the wave sources in the surrounding space resulting in constructive and destructive interferences. As one can expect, the total energy flux is conserved. However, one case of apparent nonconservation energy attracts great attention.[1,2] Imagine that a pair of coherent, point-like wave sources (located at the same position) radiates sinusoidal waves of amplitude A, spreading in a uniform medium. Assume also that radiation of the two sources is in phase. Since the energy of oscillation, E, is proportional to amplitude squared, one quickly arrives at an apparent paradox. That is, the energy of oscillation in every point due to only one source is $E_0 = CA^2$ (C is the coefficient of proportionality), while according to the linear superposition principle, the combined amplitude of oscillations from the two sources is $2A$ and the energy of oscillations is $E = C(2A)^2 = 4CA^2 = 4E_0$, i.e., four (not two) times greater than the energy of oscillation of one isolated source in the absence of the second. In the general case, superposition of two waves with identical amplitudes and wavelengths produces a wave with an intensity somewhere between zero and four times the intensity of a single wave source (depending on relative phase of the two waves). This leads to the obvious question: how can we account for the *extra* (or missing) energy that necessarily results from in-phase (or anti-phase) wave interference? This *apparent* violation of the principle of conservation energy, due to the superposition of waves, is the primary topic of this paper.

Levine[1] explained this apparent paradox by introducing wave impedance. This explanation is technically correct, although many students and teachers are not familiar with wave impedance, making it difficult for many readers to understand the given explanation. Moreover, six years after publication of Levine's paper, Mathews[2] made an attempt to explain the above apparent paradox (in the same journal as Ref. 1 but without citation of this paper). However, Mathews incorrectly states that for every region of constructive interference there will be compensating regions of destructive interference, which prevents violation of conservation of energy. To see that this is in general false, consider the simple example of two in-phase, identical wave sources at the same location. They produce constructive interference at *all* surrounding locations. These facts show that this seemingly trivial problem needs further discussion and a clear explanation.

Interference from two identical monochromatic in-phase wave sources

Let us consider interference from two identical point-like, monochromatic, and in-phase wave sources S_1 and S_2 with equal wavelengths λ (Fig. 1). Maxima and minima are found on the hyperboloids of revolution around the line containing the two sources. Conditions of maxima and minima are:

$$r_2 - r_1 = m\lambda \qquad (\max)$$

$$r_2 - r_1 = (m + \frac{1}{2})\lambda \quad (\min) \; .$$

Here $r_2 - r_1$ is the path difference of two rays and m is an integer.

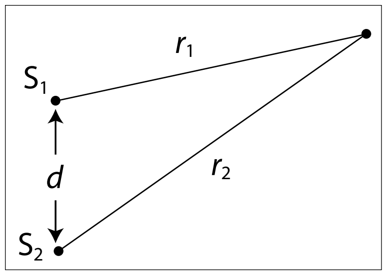

Fig. 1. Interference of two like point sources (S_1 and S_2) of monochromatic sinusoidal waves.

The total amount of energy that will be transmitted to the medium surrounding the two sources will depend on the source separation (relative to λ); phase difference between the sources may play a role as well. First let us consider two well-separated sources ($d >> \lambda$). Here, interference will be constructive in some regions and destructive in others. The total energy flux through any closed surface that encloses sources S_1 and S_2 is *nearly* equal to the sum of energy flux of two isolated sources taken one at a time. This equality, however, is *not exact*. Only as the ratio d/λ *becomes large* will the sum of the energies from two isolated sources be nearly equal to the net energy transmitted from the combined sources as noted by Refs. 3 and 4. Note also that when source separation is large (compared to λ), that the phase difference between the sources does not affect the total amount of energy transferred to the medium.

Now we consider the other extreme, $d << \lambda$ (or $r_2 - r_1 \approx 0$), then oscillations from both sources are practically in phase in all points of space. In this case the intensity of the resulting waves from the two sources in any point of the space is four (not two) times greater than the intensity of one isolated source. This is a direct result of addition of the amplitudes, which when squared to arrive at intensity, gives an increase of a factor of four. So, if $d << \lambda$, the net energy flux through any surrounding surface (which includes these two sources) is *nearly double the sum of individual energy fluxes* that these two sources would have produced *when each was isolated.*

It is important to emphasize that the law of energy conservation *does not require* the equality of power radiation of two isolated sources as compared to when they are near one another. In fact, two in-phase wave sources do generate greater

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power when close to each other ($d<\lambda$) since they now represent (effectively) a single source producing a wave of amplitude $2A$. The increase in power of radiation is not a violation of the law of energy conservation but is due to the *additional work* done by the two wave generators to produce the amplitude of ($2A$) for combined oscillations from two nearby, in-phase sources. To accomplish this increase in amplitude, the wave sources will each draw more power from their supplies. If the supplies are not capable of producing this additional power, then the combined wave amplitude will never achieve the value of $2A$ in the first place.

Equally important is the case of two closely spaced wave sources that are in anti-phase. If they were isolated, they would each produce a net energy flow to the medium, but when the two sources are placed in the same location, we expect zero energy transfer to the medium. Each of the sources will do zero work (due to the zero amplitude of the superimposed oscillations) and hence will require zero energy from whatever is driving them. Once again, there is no violation of the conservation of energy.

Interference of sound waves

Using interference of sound waves as a convenient example, the following experiments qualitatively confirm the increase (or decrease) in power of the radiation of two closely located sources in comparison to twice the power of a single isolated source.

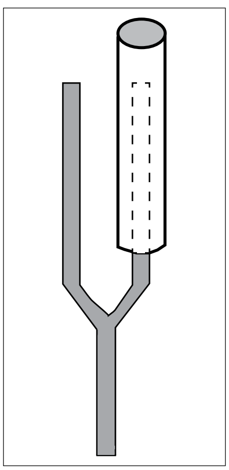

Fig. 2. A tuning fork with a cardboard cylinder on one prong produces greater intensity of sound in every point of space than a tuning fork without a cardboard cylinder.

We consider the oscillations of the tines of a simple tuning fork. The transmitted power of oscillations from a tuning fork is small primarily because the tuning fork tines oscillate *out of phase*. As the distance between the tines is much less than the wavelength, *destructive* interference is predominately the case for all points surrounding the fork. Since the tine spacing, while small, is not zero, the interference is not completely destructive (which is why we can hear a tuning fork at all). However, sound intensity is expected to be less from the typical tuning fork with two tines, as compared to one with only a single tine. This is easily confirmed by blocking the wave propagation of *one tine only* by placing it within a cardboard tube (closed at one end, see Fig. 2). This results in an increase of sound intensity, as detected by simply listening (this experiment is posted on YouTube at http://youtu.be/Cfwj8j3hNrE). This qualitatively confirms that destructive interference is the dominant case for the typical operation of a tuning fork. This point is further made by noting that the duration of the tuning fork oscillations *decreases* when the cardboard tube surrounds just one tine. Since destructive interference is prevented, *more power* is transmitted to the air, and the mechanical energy of the oscillating fork declines more quickly. Without the cardboard tube, the largely destructive interference restricts the rate of transmission of energy to the air, and thus the wave source maintains its mechanical energy for a longer time. This is instrumental in the fork's ability to produce sound for the extended period that we are accustomed to. The fact that destructive interference between two wave sources results in *longer duration* of oscillations is very counterintuitive, but is supported by the following example.

If the tines *could* oscillate in phase, the duration of oscillations would be reduced, which is the reason pianos are designed with many duplicate strings (and some cases triplicate). Sound duration of in-phase, free vibrations of multiple identical closely placed piano strings (which will generate constructive interference) is shorter than the duration of one isolated string.[5] This indicates that the power generation from a coupled in-phase pair of strings is more than twice that for a single vibrating string. The initial mechanical energy is dispersed more quickly into sound wave oscillations.

Quantitative analysis

A detailed study of interference of two loudspeakers is presented by Vanderkooy and Lipshitz.[6] The authors derived the dependence of total power of radiation of two identical isotropic loudspeakers as a function of distance between them:

$$P = 2P_0[1+\frac{\sin(kd)}{kd}\cos\theta]\,. \tag{1}$$

Here P_0 is the total (that is, summed over all directions) acoustical power of radiation of *one isolated* loudspeaker, $k=\frac{2\pi}{\lambda}$ is angular wave number, d is the speaker's spatial separation, and θ is the phase shift of oscillations between sources S_1 and S_2. Equation (1) indicates that the total power of radiation of two in-phase ($\theta = 0$) loudspeakers at the same location ($d = 0$) is four times ($P = 4P_0$) greater than the total power of one isolated loudspeaker. This fact has been confirmed experimentally by Gander and Eargle.[7] For phase shift $\theta = \pi$ (180°), and $d = 0$, the total radiation power equals zero ($P = 0$), as expected.

Now consider the case of two significantly separated sources ($kd>>1$ or $d>>\lambda$). The power of radiation is equal to twice the power of an individual (isolated) radiator ($P = 2P_0$) regardless of the phase difference, θ. Figure 3 (taken from Ref. 7) illustrates the above statements. Figure 3 confirms that the *total* integrated power of radiation depends strongly on the phase difference between sources *only* when they are closely spaced ($d \le \lambda$). For spacings larger than this, the total integrated power of two sources is always $2P_0$ (no matter what the phase difference).

When separation of sources is much less than the wavelength, the fact that total integrated power can be as large as $4P_0$ (or as small as zero) does not contradict the law of energy

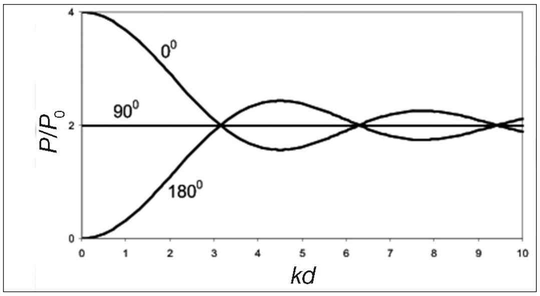

Fig. 3. Normalized (P/P_0) power of radiation of two coherent sound waves vs parameter $kd = 2\pi \frac{d}{\lambda}$ for 0°, 90°, and 180° phase shifts of oscillations of sound sources (P is the power of radiation of two sound waves and P_0 is the audible power of an isolated source of sound, i.e., P_0 is the power generated by only one source in the absence of the second with the same amplitude of oscillations as for coupled oscillations).[6]

conservation. This phenomenon is explained by the fact that the air pressure oscillations in the vicinity of one source significantly influence air pressure near the second one. In this way, a speaker's diaphragm will oscillate over a different amplitude (and thus do a different amount of work on the air) due to the proximity of the other speaker. Engebretson[8] explains these phenomena as the influence of the impedance of medium on the radiation power.

Conclusion

The superposition principle for interference of two coherent, in-phase, identical, closely located sources of waves leads to a four times increase in total power ($4P_0$) when compared to a single isolated wave source (P_0). Since the two sources are in-phase, constructive interference will occur for all points in space. However, in the case of significantly separated (but still in-phase) wave sources ($d>>\lambda$), there will exist separate regions of constructive and destructive interference, resulting in the total radiated power being two times less than for closely ($d<<\lambda$) situated sources (i.e. $2P_0$). This is not contradictory to the principle of energy conservation, however. When $d<<\lambda$, the sources of waves are coupled and will produce a greater amplitude of oscillations (and thus do more work than for well separated wave sources). For closely spaced wave sources, it is simply not possible to superimpose the oscillations from two in-phase sources without drawing more energy from the oscillators themselves. In other words, two in-phase sources will "work harder" when near one another, which solves the "paradox" of where the "extra" power comes from for this configuration. Similarly, two nearby but out-of-phase sources will "work less hard" (due to less displacement), which explains the "missing" total power.

In the equation $E_0 = CA^2$, C is proportional to

$$[1 + \frac{\sin(kd)}{kd}\cos\theta] \cdot$$

C depends on the dynamic properties of medium, which, in turn, are a function of the distance between the sources of oscillations (relative to wavelength). From Eq. (1), it is seen that only for significantly large separation of two sources does this coefficient become independent of d. Therefore, *impedance* is different for an isolated source from that of two closely situated sources. For example, consider two in-phase sources at the same location. C will be twice that of the case of one isolated source. Therefore, power generated by *each* source is twice greater than the power of a single source, and power generated by the pair of in-phase sources (in close proximity) is four times greater than power of a single isolated source. This is a mathematical explanation of the apparent paradox described at the beginning of the paper.

In general, adding the amplitudes of two identical spherical wave trains (with offset centers and arbitrary relative phase) leads to radiation patterns with a total power between zero and four times the power of one wave source alone. In the case of closely spaced centers, for which the path difference is a small fraction of the wavelength, the net power produced depends primarily on the relative phase of sources and the ratio of separation to wavelength. Finally, consider the case of widely separated centers that produce an interference pattern consisting of multiple bands of constructive and destructive interference. This condition *does* tend to result in twice the power of a single source. Ultimately, there is no paradox as to the conservation of energy during wave superposition, but the path to understanding this result is more complex than originally expected.

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