The Effect of Particle Number in N-body Simulations of the Formation of Giant Molecular Clouds

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Abstract In this paper, we study the effect of particle number in the N-body simulation of the formation of giant molecular clouds. The result shows that in the case of formation by aggregation of "unit" clouds, if we take the mean density of the unit clouds to be a constant (or their effective radius to be proportional to the cubic roots of their mass), then the disruption rate of the GMCs will be independent of the initial number of unit clouds.

Key words: ISM: clouds—ISM: evolution —Methods: N-body simulation

1. INTRODUCTION

Observations reveal that the mass of molecular clouds in galaxies is in the range of $10^3 M_\odot - 10^7 M_\odot$, and that in spiral galaxies their mass spectrum is $N(M) \propto M^{-1.6}$\textsuperscript{[1]}. In the molecular clouds the number density of $H_2$ is about $100-300 \text{cm}^{-3}$, and in their densest regions it can reach the order of $10^8 \text{cm}^{-3}$\textsuperscript{[2,3]}. When the mass of a cloud exceeds $10^5 M_\odot$, the cloud is called a giant molecular cloud (GMC). Star formation mainly takes place in GMCs, in which there are physical processes that promote the conversion of molecular matter into stars. The existence of massive stars gives rise to stellar winds, expanding HII regions, and supernova explosions, by which the GMCs are disrupted into less massive clouds. On the
other hand, collisions between clouds will lead to the formation of GMCs. Studying the recycling between the more and less massive clouds in the interstellar medium will be helpful for understanding not only the formation of GMCs itself but also the dependence of star formation rate (SFR) on it. Kwan et al.\cite{141} and Roberts et al.\cite{151} have separately studied two different mechanisms of GMC formation. The former considered collisions between clouds resulting in their coalescence, while the latter considered that when two clouds collide, only part of their kinetic energy is dissipated, and GMCs are formed from the aggregation of less massive clouds. Observations show that a great number of dense molecular clouds in our Galaxy manifest themselves as CO emission centers (peaks). Rivolo et al.\cite{111} employed the formalism of two-point covariance function to analyze the statistical clustering properties of these emission centers and presented evidence that the cores of molecular clouds are strongly clustered into groups. It can be considered as a support for the mechanism of forming GMCs by aggregation.

For studying the re-cycling of molecular clouds in the Galaxy, the method of numerical simulation is generally adopted. Limited by the power of the computer, the mass of the initial "unit" clouds taken in the simulation cannot be too small, since for a given quantity of hydrogen molecules in the system, the number of such clouds is inversely proportional to the individual mass. But there is observational evidence that there are clouds with masses as small as around 200 $M_\odot$. In current simulations, the mass of the unit cloud is generally taken as high as $10^4 M_\odot$. Hence, it has become necessary to study the effect of the number of "unit" clouds used in the simulation on the formation and evolution of GMCs. Using a series of 2-dimensional models that include the effects of cloud collisions, star formation and supernova explosions, Noguchi et al.\cite{161} have simulated the evolutionary effect of the cloud number for isolated and interacting galaxies. Their result indicated that, if the mean free path of clouds remains constant, then for both isolated and interacting galaxies the evolutionary effect is the same for different cloud numbers. This paper is an extension of this study to 3-dimensional models.

2. MODELS

In this paper three 3-dimensional models are constructed, with clouds distributed on the galactic plane to a limited height. All of the unit clouds are supposed to be spherical, and their total mass is taken as $1.2 \times 10^9 M_\odot$. At the initial moment, these unit clouds are distributed uniformly within the ring between 3 kpc and 7 kpc with a thickness of 200 pc. The initial velocity of a unit cloud is composed of two parts: the circular motion determined by the gravitational potential due to a distribution of stars including spiral perturbation, and a velocity dispersion of 10, 7.5, and 5 km/s, in the radial, longitudinal and z-directions. Under the gravitational potential the motion of clouds is described by\cite{15,9}:

$$\Psi(r, \theta, t) = \Psi_0(r)\{1 + \xi(r) \cos[2\theta - 2\Omega_p + \Phi(r)]\}, \quad (1)$$

in which $\Psi_0(r)$ is the axially symmetrical gravitational potential, i.e., the Toomre disk:

$$\Psi_0(r) = -\frac{B^2 a^3}{\sqrt{a^2 + r^2}}, \quad (2)$$
in which, $B=0.0576/\text{Myr}$; $\Omega_p$ is the pattern angular velocity, and $\Omega_p=0.0135/\text{Myr}$; $\xi(r)$ is a quantity related to the amplitude of the spiral perturbation:

$$
\xi(r) = \frac{Aa^2r^2}{5(a^2 + r^2)^2},
$$

where, $A=0.1$, $a=7\text{kpc}$. The shape of the spiral arms is given by:

$$
\Phi(r) = 2 \ln\left[1 + \left(\frac{r}{r_0}\right)^j\right]/j \tan i_0,
$$

in which $r_0=1\text{kpc}$, $i_0=20^\circ$, $j=5$. The effective range of the self-gravitation between the clouds is taken as $235\text{pc}$. We shall be considering two different models in which the mass of the unit cloud is taken respectively as $10^4M_\odot$ and $10^5M_\odot$. It means that the total number of unit clouds within the molecular ring is either 120000 or 12000. Collision between two clouds is taken to be inelastic with coefficient of restitution 0.3 along the line joining their centers, and to be perfectly elastic in the perpendicular direction. Thus, collisions between clouds dissipate off their relative kinetic energy, and under the action of self-gravitation the clouds are aggregated together to form larger clouds.

The aggregation of clouds depends on their collision cross-section. Generally, there are two ways to determine this parameter. The first is to assume that all the clouds have the same mean density, i.e., their mass is proportional to the third power of their geometric radius. Observations show that the mean density of molecular clouds is in the range of 100 to 300 cm$^{-3}$. If, following Kahn et al.\cite{[9]}, the mean density is taken to be $200\text{cm}^{-3}$, then the geometric radii of unit clouds with masses $10^4M_\odot$ and $10^5M_\odot$ will be 5.85 pc and 12.6 pc, respectively. Due to the existence of the magnetic field, the effective radius of the cloud is two times its geometric radius\cite{[10]}. Hence, for the unit clouds with the above masses, the effective radii will be 11.7 pc and 25.20 pc, respectively. The second way to define the collision cross section is to assume that all clouds have the same mean free path, i.e., the cloud mass is proportional to the square of the effective radius. In this paper we follow the first way.

The mean free path is given by the following formula:

$$
\lambda = (\sqrt{2n}\Sigma)^{-1},
$$

in which $n$ is the mean number density of clouds, and $\Sigma$ is their collision cross-section. For the cloud with mass $10^4M_\odot$, its effective radius is 11.7 pc, then from formula (5) its mean free path is 86.1 pc. If, on the other hand, we assumed that the clouds have the same mean free path, then from formula (5) the effective radius of the unit clouds with mass $10^5M_\odot$ will be 37.0 pc.

Collisions between clouds give rise to clusters of clouds, leading to GMCs. The identification of GMCs is effected using a percolation program. For the unit clouds with mass $10^4M_\odot$, a percolation parameter (i.e., cluster scale) of 30 pc is adopted, which is equal to 2.564 times the effective radius. The same ratio (2.564) of percolation parameter to effective radius is taken for the other case. Three models, labelled M, J and Q are considered in this paper. Their parameters are listed in Table 1.
Table 1  Computational models

<table>
<thead>
<tr>
<th>Model</th>
<th>Number</th>
<th>Mass($M_\odot$)</th>
<th>Geometrical radius</th>
<th>Effective radius</th>
<th>Cluster scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>120000</td>
<td>$10^4$</td>
<td>5.85</td>
<td>11.7</td>
<td>30.0</td>
</tr>
<tr>
<td>J</td>
<td>12000</td>
<td>$10^5$</td>
<td>12.60</td>
<td>25.2</td>
<td>64.6</td>
</tr>
<tr>
<td>Q</td>
<td>12000</td>
<td>$10^5$</td>
<td>18.85</td>
<td>37.0</td>
<td>94.87</td>
</tr>
</tbody>
</table>

Star formation disrupts the GMCs. This process is modelled in this paper as follows. When a GMC is identified by the percolation program, if its mass is greater than or equal to $10^5 M_\odot$, then it is considered possible for star formation to take place and for this GMC to be disrupted. Let the probability of disruption of a GMC with a mass greater than or equal to $10^5 M_\odot$ be:

$$P(M) = \frac{1}{\alpha} \left[ 1 + \log\left( \frac{M}{10^5 M_\odot} \right) \right],$$

in which $M$ is the mass of the GMC, and $\alpha$ is the coefficient of disruption (in this paper $\alpha=40$). When an identified GMC is confirmed to be disrupted, its constituent clouds will be redistributed randomly within the whole molecular ring. At their new positions, these clouds will get their new initial velocities that are composed of the local circular motion due the axially symmetrical gravitational potential and peculiar motions according to the velocity dispersions stated above. This process supplies the clouds with the kinetic energy dissipated in collisions.

A time step length $5 \times 10^5$ yr is adopted in our simulation, so as not to miss important collisions pairs while limiting ourselves to reasonable CPU times. At every step colliding pairs will be sorted out. In every 5 Myr, disrupted GMCs will be sorted out, and the unit clouds in them will be relocated in the manner described above. The total time of simulation is $6 \times 10^5$ yr.

3. RESULT

For evolution times 350, 400, 450, 500, 550, and 600 Myr, the mass spectrum and the ratio function $F = F(M)$ are shown in Fig. 1 (Model M) and Fig. 2 (Model J). The ratio function $F(M)$ is the ratio of the total mass of all identified GMCs with masses greater than $M$ to the total mass of all the clouds. From the general behavior of this function shown, we see that the distribution of clouds is almost independent of the evolution time after 300 Myr. In other words, after that time, a relatively steady state is reached between the disruption by star formation and the aggregation from inelastic collisions. Accordingly, our numerical simulation is terminated at 600 Myr. The figures also demonstrate that the ratio function of Model M is different from that of Model J. In fact, the ratio function of Model J is similar to that obtained by Kwan et al.\[4\], i.e., it is downward convex all the way. Kwan et al. adopted a different mechanism of GMC formation from us but used the same unit cloud mass of $10^5 M_\odot$. In our models (Models M, J, Q), the relative fraction of massive GMCs is small, and the mass spectrum is high for masses less than $10^{2.6} M_\odot$ in Model M and for masses less than $10^{5.6} M_\odot$ in Models J and Q. This tendency is kept up from 350 Myr to 600 Myr. Since we are concerned with the formation of massive GMCs, so we just make a linear fitting to
Fig. 1 The ratio function (left panel) and the mass spectrum (right panel) in Model M at various times between 350 and 600 Myr. SPIRAL indicates inclusion of the effect of spiral perturbation, "Whole" means that after disruption the clouds are redistributed within the whole molecular ring, log(40) indicates the coefficient of disruption \(\alpha=40\).

The above result is obtained on the assumption that all the unit clouds in the different models have the same mean density, and hence their masses are proportional to the third power of their effective radii.

The above analysis demonstrates that from 300 Myr after the start of the evolution, the distribution of clouds in either Model M or Model J is almost time independent, i.e., a relatively steady state is reached between the disruption of GMCs caused by star formation and the aggregation of clouds caused by inelastic collisions. Accordingly we can calculate the disruption rate of GMCs shown in Table 2.

Table 2 shows that the average disruption rates of GMCs is basically the same in Model M and Model J (4.566 and 4.667 \(M_\odot/yr\)). If star formation efficiency is taken as 30%\({}^{[11]}\) for those GMCs in which star formation is confirmed, then the rate of star formation is about 1.38 \(M_\odot/yr\) for both Model M and Model J. Thus it is found that the number of unit clouds has no effect on the rate of star formation, hence on the rate of disruption of GMCs.
Fig. 2 The same as Fig.1, but for Model J

Table 2 Average disruption rate

<table>
<thead>
<tr>
<th>Model</th>
<th>Average disruption rate ($M_\odot$/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>4.566</td>
</tr>
<tr>
<td>J</td>
<td>4.667</td>
</tr>
<tr>
<td>Q</td>
<td>11.628</td>
</tr>
</tbody>
</table>

Model Q is built on the assumption that the mean free path is kept constant irrespective of the number of unit clouds, i.e., the effective radius of the unit clouds is proportional to the square root of its mass. In Model Q the number of unit clouds is taken to be the same as in Model J. In order to keep the same mean free path as in Model M, the effective radius of the unit clouds is taken to be 37.0 pc. Fig.3 shows the mass spectrum and ratio function of clouds in Model Q. The ratio function shows that it continues to evolve after 300 Myr, and the distribution of cloud is not independent of the time of evolution. This means that a relatively steady state was not reached between the disruption and the aggregation. In addition, Table 2 shows that the average disruption rate of GMCs in Model Q is $11.62 M_\odot$/yr, much greater than in Model M or Model J. Apart from the fact Model Q has a spectral index of mass spectrum not very different from that of Model J, the two models are very different in all respects. This is mainly because that, compared to Model J, in Model Q the clouds have a larger effective radius and a shorter mean free path. For the same velocity dispersion, the mean collision frequency is higher in Model Q than in Model J, and the time required to reach a relatively steady state is longer. It can be concluded therefore that in the case of constant mean free path, if we start the simulation with a smaller number of unit clouds, then we shall have a much higher star formation rate, and there will be an obvious effect of the number of initial clouds.
4. DISCUSSION

Dictated by computer capability, numerical simulations are generally made with a limited number of initial, unit clouds. Basically the result of calculations can explain the most of the observational facts. It is very important to decrease as much as possible the number of unit clouds (i.e., to increase their mass), so that the amount of calculation can be reduced. It is found that if all the unit clouds have the same mean density, then after 300 Myr of evolution, the system will reach a relatively steady state in which the disruption of GMCs by star formation is balanced by the aggregation of unit clouds into GMCs. The star formation rate strictly depends on the disruption rate of GMCs and the larger the mass of the GMC, the higher is the star formation rate.

As shown above, for a fixed total mass of clouds and different numbers of unit clouds, the assumption that the effective radius of the unit cloud is proportional to the cubic root of its mass will lead to consistent results. If the effective radius of the unit clouds is taken to be proportional to the square root of its mass, then although in the different models the mean free path is the same, the rate of formation of GMCs will be different, and hence the rate of star formation will also be different. Fig. 4 gives, for Model M, the relations between the mass $M$ of a GMC at 450 Myr and its maximum diameter $D_{\text{max}}$ and its maximum height $D_z$ in the z-direction. It can be seen that the GMCs formed by aggregation are flattened objects, and that between the mass $M$ and $D_{\text{max}}$, the relation $M \propto (D_{\text{max}})^2$ holds. $D_z$ is much smaller than $D_{\text{max}}$, and it increases with $M$ very slowly. A comparison between Model M and Model Q indicates that if the number of unit clouds is decreased, (or the mass of unit clouds is increased,) while their shape remains spherical, then to keep the mean free path constant the effective radius of the unit clouds should be proportional to the square
root of their mass. Then more collisions among massive unit clouds will take place in the direction perpendicular to the galactic disk, and massive GMCs will be easier to form. And the comparison between Model M and Model J indicates that if the effective radius of the unit clouds is proportional to the square root of their mass, then collisions among the clouds will lead to a formation efficiency of GMCs equivalent to the model of less massive unit clouds. The models built by Noguchi and Ishibashi[5] are idealized 2-dimensional models. In the course of evolution, all GMCs have the same mass, and collision radius has no influence on the formation of GMCs in which star formation is possible. As the number of clouds is changed, the mean free path is kept constant. Although the collision radius influences on the collisions among clouds, its effect is only manifested as a difference of star formation rate if star formation is caused by collisions among clouds, they believe that the number of clouds has no influence on the result of evolution. This is obviously different from this paper. Our analysis demonstrates that as soon as the GMC forms, its formation efficiency will depend on the collisions among clouds, i.e., on the collision radius.

5. CONCLUSION

In the 3-dimensional simulation for the formation of GMCs by aggregation, a decrease in the number of unit clouds (or an increase in the mass of the unit clouds) will not give consistent results if their effective radius is taken as proportional to the square root of their mass. While GMCs formed by collisions of lesser clouds are flattened systems, the unit clouds in a model of more massive unit clouds are spherical, so if the mean free path is kept constant, then there will be an increase in the collisional frequency. Therefore, in 3-dimensional models with different masses of unit clouds, the effective radius of the unit cloud should be proportional to the cubic root of its mass.
References

3 Myers P. C., ibid.